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Total Gravitational Force Re-cast in Complex Space-Time Frame

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Abstract: Einstein's general theory of relativity provoked the need to have an experiment if it had to be considered valid. As a result, Gravity Probe-B experiment was suggested and conducted. The results of the GP-B experiment have shown that the general theory of relativity was correct but it is limited. More information was revealed from this experiment than the earlier predictions. Several theoretical researches have been attempted to develop a single model that puts together the predictions or GR and the experimental result. This has not happened. A major setback in the numerous studies towards obtaining such a model has been the lack of a robust and well elaborate mathematical structure. This study has therefore identified a comprehensive mathematical structure to re-cast the predictions and also determine the finer details of the extra experimental data with accuracy. The new mathematical structure re-defines the four vector frame by expressing the fourth time axis as a vector just like the three well known spatial axes. However, the time axis is assumed to take a general orientation as opposed to the spatial axes which are mutually orthogonal. The unit vector on the time axis is complex and it is represented by the temporal unit vector $\hat{k}$.The four-vector mathematics in this complex plane agrees with the three-dimensional vector mathematics that is well established. This study therefore intends to comprehensively reformulate geodetic effect and frame-dragging effect.

Keywords: Gravitational Force, Gravitational Field, Four Vector Complex Frame

1. Introduction

The general theory of relativity predicted both geodetic effect and frame-dragging effect. Since it was a theoretical model, an experiment was suggested later on to validate these predictions [1]. After many years, a NASA sponsored experiment; GP-B did validate the findings. The findings of the experiment brought on board fresh challenges. One of the challenges was that the theoretical general theory of relativity was correct but limited. It was not exhaustive in itself because the experiment presented more information than the earlier predictions. Numerous researches have been done as a result of the experimental findings, so far it has remained a problem to get a single theoretical model to completely and exhaustively account for the predictions and the experimental data [2, 6, 10, 12].

The biggest problem has been the lack of a relevant mathematical structure to be used. There has been a weakness in linking the idea of a curved space-time, tensors and physics. This has not been done carefully from first principles. In the intended research, a comprehensive and rigorous mathematical structure has been identified which will re-cast geodetic effect, frame-dragging effect and at the same time account for the extra experimental information of the gravity probe-B experiment. In this study, a more comprehensive mathematical structure that can re-cast geodetic effect, frame dragging effect and account for the extra information will be identified and used. The three important identified outcomes of the GP-B experiment will be determined in a single calculation. The new mathematical structure to be used is the Euclidean four dimensional complex reference frame [13].

The main aim of this research is to comprehensively establish a theoretical model that will account for the predictions of the general theory of relativity and at the same time determine and account for the extra findings of GP-B experiment.

It is assumed that the spatial axes of the new complex frame are mutually orthogonal and the temporal time axis will be considered to take a general orientation. The research changes the way physicists have perceived general relativity. How? It will simplify the abstract complex way in which it has been presented over a long time. It will comprehensively re-cast the general theory of relativity in a new fashion and finally determine and account for the limitation it has carried for a very long time. There have been numerous attempts by physicists to study general relativity and link it to the experimental results. This has not happened very clearly. It has consistently been unsuccessful because at times some unclear corrections have been imposed. The approximations and corrections that have been used to stand in for the extra data will be clearly calculated and explained. Hence their nature and values will be known.

The research methodology will mainly be model building. Geodetic effect and frame dragging effect have been studied extensively by both theoretical models and experimental models. These effects are predictions of the general theory of relativity. The studies have been conducted in the four-dimensional framework where there are three spatial vectors and a time axis that is hanging. The time axis is hanging because it has not been precisely explained or shown whether it is a vector or a scalar. This study clearly presents the time axis as a vector that takes a general orientation compared to the three spatial axes which are mutually perpendicular to each other. So the time axis which is also called the temporal axis can either be perpendicular to the spatial axes or not.

Geodetic effect and Frame-Dragging effect are among the predictions of the general theory of relativity. The research is a study of geodetic effect and Frame-Dragging effect in an extended four-dimensional complex vector frame. In this study it is assumed that more accurate expressions and values of these effects will be obtained. The new mathematical structure involves the time axis that has been re-formulated into a vector just like the three spatial axes. The temporal axis will now be represented by a complex unit vector $\hat{k}$ .Usually a four-vector has been represented as $A = (\emptyset, \vec{A})$. This study will alter this representation to $A = -ic\emptyset\hat{k} + \vec{A}$.The current representation has limited operations. For example it has not been possible to determine the curl of a vector step by step. Instead, the curl of a vector has been determined in terms on components. With the new representation of a four-vector, the curl of vector will now be defined step by step from first principles. The four-vector will now be re-defined research will add value by redefining the fourth time axis as a vector as; $P = -ic\rho\hat{k} + \vec{P} = -ic\rho\hat{k} + P_x\hat{x} + P_y\hat{y} + P_z\hat{z}$. The fourth time axis is defined as a complex vector with unit vector $\hat{k}$. The resultant frame has a complex axis, so it is a complex frame. A new model will therefore be built based on this enhanced re-definition of the fourth time axis. This will be applied on all aspects of dynamics that will be discussed in this research.

Hence the study in general will be a reformulation of geodetic effect and frame-dragging effect in a new mathematical frame-work in which the fourth time axis is a vector. This study therefore aims at improving the accuracy and the predictions of the general theory of relativity. Computer simulation will be applied finally.

General relativity is a theory of gravitation presented in tensor notation. It is based on the assumption that observed gravitational effects are caused by the warping of space-time. General relativity is a relativistic theory of gravity among other theories. It is considered as the simplest. Our task in this research is to present an even simpler model using vectors.

The general theory of relativity was developed in a series of papers by Albert Einstein. This theoretical model predicted two effects; the geodetic precession and the frame dragging precession. In 1959, George Pugh suggested an experiment to confirm the predictions. Independently, in 1960, Leonard Schiff also suggested the same experiment [1]. The gravity probe –B experiment that was conducted by NASA at the Stanford University confirmed the predictions of the general theory of relativity [2]. Several authors have reported that the confirmation was by no means very high [1, 3, 4, 5, 6].

Frame-dragging effect is at times called Lense-Thirring effect. It is a relativistic effect that is caused by the rotation of the earth. It has also been measured by laser ranging to two earth satellites [7]. According to [7, 8], this is currently the most accurate value of frame-dragging.

De Sitter is also a frame dragging effect [10]. In this case, the earth and the moon form a system that can be viewed as a single body moving in the gravitional field of the sun. So the earth-moon system is a gyroscope which is rotating in the gravitational field of the sun. It is the frame dragging effect of the earth-moon system that has been measured using laser ranging [10]. Geodetic effect is implicit in the relativistic equations of motion [11]. The outcome of the GP-B experiment raised an issue. The issue was that general theory of relativity failed to predict some information. Although its predictions were accurate, the general theory is not exhaustive in itself. It is limited because the experimental data contained more information than the predictions of the general theory of relativity [12]. Several researches have been attempted to come up with a more exhaustive theoretical model that includes both the predictions of the general theory of relativity and the extra GP-B experimental data. There has been no success [2, 12]. The main problem why this has not worked is the limitations of the mathematical models that have been used.

This research has identified a comprehensive mathematical structure [13] which can in a robust way recast all the predictions and at the same time neatly determine the extra experimental data without any approximations and corrections. The new mathematical structure re-defines the time axis presenting it as a vector. It therefore forms the fourth axis having a complex unit vector. The other three spatial axes remain as before being mutually orthogonal. The complex time axis will be considered to take a general orientation in this study.

Mathematical operations in this new frame fully obey all vector operations in the well established 3-D vector mathematics [13]. This four vector mathematics fully and elegantly suggest a straight forward way in which dynamics can be studied in a four vector frame [14]. This is the motivation behind this study.

In 1905, the special theory of relativity was published by Einstein. After that, in 1907 he used a thought experiment of an observer in free fall to further his study. It took eight years to get what is today called Einstein field equations. They specify how geometry of space and time is influenced by matter. The problem that has persisted is that these equations are non-linear and extremely difficult to solve and therefore to be understood. It is these equations that form the core of the general theory of relativity. In 1917, Einstein applied this theory to the universe. Assuming that the universe was static, he added a new parameter called the cosmological constant. This new theory was superior to Newtonian gravity and was consistent with the special theory of relativity. It clearly accounted for several field effects that had been cited and explained by Newtonian theory.

Very tiny and light particles which are considered to be somehow mass-less to the extend that their own gravitational effect is zero are used in an experiment. In the absence of gravity and any other external forces, these test particles move in straight lines at constant speed. So they move along straight world lines in space time. In the presence of gravity, the particles move along lines called geodesics, curved lines. So in such a case, space-time is non-Euclidean, it is curved. In special relativity, parallel geodesics remain parallel. Gravity makes space-time to curve. The straight time-like lines of free fall are therefore made to be deformed to get curved ones when gravity is considered. Hence it must therefore mean that a change in space-time geometry becomes necessary. The mathematics of curved lines is tensors. Most people are not comfortable with tensors. This means that everything must change to the curved generalization of Minkowski space.

Special relativity is only suitable in cases where there is no gravity. It is also only valid in inertial frames. Since we do not have global inertial frames, it becomes impossible to use it readily. The Einstein field equations are non-linear and very difficult to solve. The process of linearization of these field equations is equally very involving. This is the motivation behind our interest in trying an alternative that is straight forward and easy to follow. It is from this background that we draw the courage to develop an alternative approach to field theory so that issues like gravity can be addressed by use of vectors. Now the big problem is the nature of the field equations and the manner in which they are obtained.

2. Computations and Derivations

An electromagnetic field is a four vector and can just be compared to gravitational field around a large mass. There are forces in an electromagnetic field which are comparable to forces in a magnetic field. The forces in the two fields are four vectors each having a complex part to add on the known three space components in ordinary vector mathematics.

Maxwell's field equations were formulated in the three dimensional frame-work. For example, $\nabla . E = \rho$. It means that the divergence of an electric field is the charge density. In three dimensional space the field in question spreads out in the x direction, y direction and z direction. The time direction is not considered. The divergence of the magnetic field is zero because there are no known magnetic monopoles. So the divergence of the magnetic field is zero, $\nabla . B = 0$.

Maxwell's equations have always been written and considered in three dimensions.

$$\vec{\nabla} . \vec{E} = \rho \tag{1}$$

$$\vec{\nabla} . \vec{B} = 0 \tag{2}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3}$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{J} \tag{4}$$

Re-writing them in four dimension complex form, we have

$$\nabla . E = \rho \tag{5}$$

$$\nabla . B = 0 \tag{6}$$

$$\nabla \times E = \frac{\partial B}{\partial t} \tag{7}$$

$$\nabla \times B = -\frac{\partial E}{\partial t} + J \tag{8}$$

which in a more explicit form are written as

$$\frac{\partial E_k}{\partial k} + \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \rho \tag{9}$$

$$\frac{\partial B_k}{\partial k} + \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \tag{10}$$

$$\hat{k} \times \left(\frac{\partial}{\partial k} E_x \hat{x} + \frac{\partial}{\partial k} E_y \hat{y} + \frac{\partial}{\partial k} E_z \hat{z}\right) + \left(\frac{\partial}{\partial x} E_k \hat{x} + \frac{\partial}{\partial y} E_k \hat{y} + \frac{\partial}{\partial z} E_k \hat{z}\right) \times \hat{k} + \vec{\nabla} \times \vec{\mathrm{E}} = \frac{\partial B}{\partial t} \tag{11}$$

$$\hat{k} \times \left(\frac{\partial}{\partial k} B_x \hat{x} + \frac{\partial}{\partial k} B_y \hat{y} + \frac{\partial}{\partial k} B_z \hat{z}\right) + \left(\frac{\partial}{\partial x} B_k \hat{x} + \frac{\partial}{\partial y} B_k \hat{y} + \frac{\partial}{\partial z} B_k \hat{z}\right) \times \hat{k} + \vec{\nabla} \times \vec{\mathrm{B}} = -\frac{\partial E}{\partial t} + J \tag{12}$$

And finally we will also write the Lorentz equation in four dimension form

$$F = e\left[E + \left(\hat{k} \times \left(v_k B_x \hat{x} + v_k B_y \hat{y} + v_k B_z \hat{z}\right) + \left(v_x \hat{x} B_k + v_y \hat{y} B_k + v_z \hat{z} B_k\right) \times \hat{k} + \left(v_y B_z - v_z B_y\right)\hat{x} + \left(v_z B_x - v_x B_z\right)\hat{y} + \left(v_x B_y - v_y B_x\right)\hat{z}\right)\right] \tag{13}$$

The equations (9), (10), (11) and (12) are the Maxwell's equations in four dimension complex frame. Equation (13) is the Lorentz equation in four dimension complex frame. To get field equations, all we need is a lagrangian to subject it to the principle of least action. The result will be differential equations of motion which will be field equations in the complex space time frame.

In an electromagnetic field, there is a magnetic field coupled with an electric field. Ordinarily, two types of forces are discussed in this field. The electric field $\vec{E}$ and the magnetic field $\vec{B}$. For a long time to date, these two forces have discussed and obtained independently.

The electric force

$$\vec{F}_e = e\vec{E} \tag{14}$$

and the magnetic force

$$\vec{F}_m = e\vec{v} \times \vec{B} \tag{15}$$

Because of relativistic corrections, the magnetic force is adjusted to become

$$\vec{F}_m = e\frac{\vec{v}}{c} \times \vec{B} \tag{16}$$

Finally, the total force in an electromagnetic field is the sum of the two forces.

$$\vec{F} = e\frac{\vec{v}}{c} \times \vec{B} + e\vec{E} = \frac{\vec{J}}{c} \times \vec{B} + e\vec{E} \tag{17}$$

Hence electromagnetic field is therefore a two component force. In this paper, we deviate from this kind of treatment and come up with a new version to discuss the same. We develop a new and rigorous calculation to obtain the same result in a single calculation.

We begin by defining what a field is and how a field comes about. A field is created by a mass or a charge.

The curl of a four vector is done by removing the brackets step by step.

$$\nabla \times A = \left(\frac{i}{c}\frac{\partial}{\partial t}\hat{k} + \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) \times \left(A_k\hat{k} + A_x\hat{x} + A_y\hat{y} + A_z\hat{z}\right) \tag{18}$$

$$\nabla \times A = (\vec{\nabla} \times \vec{A})_x\hat{x} + (\vec{\nabla} \times \vec{A})_y\hat{y} + (\vec{\nabla} \times \vec{A})_z\hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial x}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial x}A_k\right)\hat{z}\right\} \times \hat{k} \tag{19}$$

$$= \vec{\nabla} \times \vec{A} + i\left(-\frac{1}{C}\frac{\partial}{\partial T}\vec{A} - \vec{\nabla}(c\emptyset)\right) \times \hat{k} \tag{20}$$

From standard knowledge in electrodynamics, we note that

$$\vec{B} = \vec{\nabla} \times \vec{A} \text{ and } \vec{E} = -\frac{1}{C}\frac{\partial}{\partial T}\vec{A} - \vec{\nabla}(c\emptyset) \tag{21}$$

Therefore for the case of an electromagnetic field, the result is interpreted as

$$\vec{F} = \vec{B} + i\vec{E} \times \hat{k} \tag{22}$$

Equation (17) and equation (22) present the same idea in which two forces are added to give a resultant force which is a two component force; one is a magnetic force and the other is the electric force. The unique thing in that the two situations are exactly identical. One is obtained by adding two independent forces while the other is obtained just directly from one single calculation. This approach of adding two independent forces is well known and well established. This paper now presents a new approach that gives the same result from only a single calculation. We go ahead and transfer the case from an electromagnetic field to a gravitational field. For a gravitational force, the electric force is replaced by the Newtonian gravitational force and the magnetic force remains as such. This means the gravitational force is also a two component force just like the electromagnetic force. A charge causes an electric field which causes are magnetic forces and electric forces. A large mass causes a gravitational field which causes magnetic forces and the Newtonian gravitational forces. The two systems are therefore identical.

Since both the electromagnetic field and the gravitational field are four vectors, it follows from the above argument that

$$\nabla \times A = B_x\hat{x} + B_y\hat{y} + B_z\hat{z} - i\left(g_x\hat{x} + g_y\hat{y} + g_z\hat{z}\right) \times \hat{k} \tag{23}$$

This can now be re-written in a familiar result as

$$\nabla \times A = \vec{B} + i(\vec{g}) \times \hat{k} \tag{24}$$

where $\vec{B}$ is the magnetic force and $\vec{g}$is the gravitational force.

This resembles the Lorentz type of force that is well known in electrodynamics. It represents a force

$$\vec{F} = \nabla \times A = \vec{B} + i(\vec{g}) \times \hat{k} \tag{25}$$

In this case, we take A as the general complex four-vector gravitational field intensity. Hence the curl of four vector gravitational field intensity A generates a two component force$\vec{F}$. This is the gravitational force. The first component is the gravitational field induction $\vec{B}$ that causes rotation in the gravitational field. The second component $i(\vec{g}) \times \hat{k}$ represents the familiar Newtonian gravitational field intensity that causes translation in the field. It now makes sense to compare an electromagnetic field with a gravitational field. It is what is called gravito-electromagnetism.

3. The General Gravitational Field Force

The curl of a vector is another vector. The gravitational force is a four vector. Let us then get the cross product between a four vector and the gravitational force derived above. Here we take the four vector to be the source current density four-vector $\vec{q}$. The resultant four-vector is what we call in this paper the general gravitational field force. So the general gravitational field force is denoted by $\vec{f}$

The source current density is a four vector q which is represented as

$$q = q_k\hat{k} + q_x\hat{x} + q_y\hat{y} + q_z\hat{z} = q_k\hat{k} + \vec{q} \tag{26}$$

$$q_k = -ic\rho \tag{27}$$

$$i = \sqrt{-1} \tag{28}$$

$$\vec{f} = q \times (\nabla \times A) = \left(q\hat{k} + q\hat{x} + q\hat{y} + q\hat{z}\right) \times \left\{B_x + B_y + B_z - i\left(g_x\hat{x} + g_y\hat{y} + g_z\hat{z}\right) \times \hat{k}\right\} \tag{29}$$

$$= \left(q_k\hat{k} + q_x\hat{x} + q_y\hat{y} + q_z\hat{z}\right) \times \left[\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{x} + \left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{y} + \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} \times \hat{k}\right] \tag{30}$$

$$= \left(q_k\hat{k} + q_x\hat{x} + q_y\hat{y} + q_z\hat{z}\right) \times \left[\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{x} + \left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{y} + \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} \times \hat{k}\right] \tag{31}$$

$$= q_k\hat{k} \times \left[\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{x} + \left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{y} + \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} \times \hat{k}\right] + q_x\hat{x} \times \left[\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{x} + \left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{y} + \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} \times \hat{k}\right] + q_y\hat{y} \times \left[\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{x} + \left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{y} + \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} \times \hat{k}\right] + q_z\hat{z} \times \left[\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{x} + \left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{y} + \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} \times \hat{k}\right] \tag{32}$$

$$= \left[q_k\hat{k} \times \left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{x} + q_k\hat{k} \times \left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{y} + q_k\hat{k} \times \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} q_k\hat{k} \times\times \hat{k}\right] + \left[q_x\hat{x} \times \left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{x} + q_x\hat{x} \times \left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{y} + q_x\hat{x} \times \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} q_x\hat{x} \times \hat{k}\right] + \left[q_y\hat{y} \times \left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{x} + q_y\hat{y} \times \left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{y} + q_y\hat{y} \times \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} q_y\hat{y} \times\times \hat{k}\right] + \left[q_z\hat{z} \times \left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{x} + q_z\hat{z} \times \left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{y} + q_z\hat{z} \times \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} q_z\hat{z} \times \hat{k}\right] \tag{33}$$

$$= \left[q_k\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{k} \times \hat{x} + q_k\left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{k} \times \hat{y} + q_k\left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{k} \times \hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} q_k\hat{k} \times\times \hat{k}\right] + \left[q_x\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{x} \times \hat{x} + q_x\left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{x} \times \hat{y} + q_x\left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{x} \times \hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} q_x\hat{x} \times \hat{k}\right] + \left[q_y\hat{y} \times \left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{x} + q_y\hat{y} \times \left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{y} + q_y\hat{y} \times \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} q_y\hat{y} \times\times \hat{k}\right] + \left[q_z\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{z} \times \hat{x} + q_z\left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{z} \times \hat{y} + q_z\left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{z} \times \hat{z} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} q_z\hat{z} \times \hat{k}\right] \tag{34}$$

$$= \left[q_k\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{k} \times \hat{x} + q_k\left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{k} \times \hat{y} + q_k\left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{k} \times \hat{z}\right] + \left[q_x\left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{z} - q_x\left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{y} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} q_x\hat{x} \times \hat{k}\right] + \left[-q_y\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{z} + q_y\left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{x} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} q_y\hat{y} \times\times \hat{k}\right] + \left[q_z\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{y} - q_z\left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{x} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} q_z\hat{z} \times \hat{k}\right] \tag{35}$$

$$= \left[q_k\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{k} \times \hat{x} + q_k\left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{k} \times \hat{y} + q_k\left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{k} \times \hat{z}\right] + \left[q_x\left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{z} - q_x\left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{y} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} q_x\hat{x} \times \hat{k}\right] + \left[-q_y\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{z} + q_y\left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{x} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} q_y\hat{y} \times\times \hat{k}\right] + \left[q_z\left(\frac{\partial}{\partial y}A_Z - \frac{\partial}{\partial z}A_y\right)\hat{y} - q_z\left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{x} - i\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}A_x - \frac{\partial}{\partial x}A_k\right)\hat{x} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_y - \frac{\partial}{\partial y}A_k\right)\hat{y} + \left(\frac{1}{c}\frac{\partial}{\partial t}A_z - \frac{\partial}{\partial z}A_k\right)\hat{z}\right\} q_z\hat{z} \times \hat{k}\right] \tag{36}$$

$$= \left[q_kB_x\hat{k} \times \hat{x} + q_kB_y\hat{k} \times \hat{y} + q_kB_z\hat{k} \times \hat{z}\right] + \left[q_xB_y\hat{z} - q_xB_z\hat{y} - i\{g_x\hat{x} + g_y\hat{y} + g_z\hat{z}\}q_x\hat{x} \times \hat{k}\right] + \left[-q_yB_x\hat{z} + q_yB_z\hat{x} - i\{g_x\hat{x} + g_y\hat{y} + g_z\hat{z}\}q_y\hat{y} \times \hat{k}\right] + \left[q_zB_x\hat{y} - q_zB_y\hat{x} - i\{g_x\hat{x} + g_y\hat{y} + g_z\hat{z}\}q_z\hat{z} \times \hat{k}\right] \tag{37}$$

Therefore this is the general gravitational force field.

$$generalforce = \vec{f} = q \times (\nabla \times A) = \left[q_k B_x \hat{k} \times \hat{x} + q_k B_y \hat{k} \times \hat{y} + q_k B_z \hat{k} \times \hat{z}\right] + \left[q_x B_y \hat{z} - q_x B_z \hat{y} - i\{g_x \hat{x} + g_y \hat{y} + g_z \hat{z}\} q_x \hat{x} \times \hat{k}\right] + \left[-q_y B_x \hat{z} + q_y F_z \hat{x} - i\{g_x \hat{x} + g_y \hat{y} + g_z \hat{z}\} q_y \hat{y} \times \hat{k}\right] + \left[q_z B_x \hat{y} - q_z B_y \hat{x} - i\{g_x \hat{x} + g_y \hat{y} + g_z \hat{z}\} q_z \hat{z} \times \hat{k}\right] \quad (38)$$

$$= q_x B_y \hat{z} - q_x B_z \hat{y} + -q_y B_x \hat{z} + q_y B_z \hat{x} + q_z B_x \hat{y} - q_z B_y \hat{x} + \left[q_k B_x \hat{k} \times \hat{x} + q_k B_y \hat{k} \times \hat{y} + q_k B_z \hat{k} \times \hat{z}\right] - i\left[\{g_x \hat{x} + g_y \hat{y} + g_z \hat{z}\} q_x \hat{x} \times \hat{k} + \{g_x \hat{x} + g_y \hat{y} + g_z \hat{z}\} q_y \hat{y} \times \hat{k} + \{g_x \hat{x} + g_y \hat{y} + g_z \hat{z}\} q_z \hat{z} \times \hat{k}\right] \quad (39)$$

$$= q_x B_y \hat{z} - q_x B_z \hat{y} + q_y B_z \hat{x} - q_y B_x \hat{z} + q_z B_x \hat{y} - q_z B_y \hat{x} + \left[q_k B_x \hat{k} \times \hat{x} + q_k B_y \hat{k} \times \hat{y} + q_k B_z \hat{k} \times \hat{z}\right] - i\left[\{g_x \hat{x} + g_y \hat{y} + g_z \hat{z}\} q_x \hat{x} \times \hat{k} + \{g_x \hat{x} + g_y \hat{y} + g_z \hat{z}\} q_y \hat{y} \times \hat{k} + \{g_x \hat{x} + g_y \hat{y} + g_z \hat{z}\} q_z \hat{z} \times \hat{k}\right] \quad (40)$$

$$= \left(q_x B_y \hat{z} - q_x B_z \hat{y} + q_y B_z \hat{x} - q_y B_x \hat{z} + q_z B_x \hat{y} - q_z B_y \hat{x}\right) + \left[q_k B_x \hat{k} \times \hat{x} + q_k B_y \hat{k} \times \hat{y} + q_k B_z \hat{k} \times \hat{z}\right] - i\left[\{g_x \hat{x} + g_y \hat{y} + g_z \hat{z}\} q_x \hat{x} \times \hat{k} + \{g_x \hat{x} + g_y \hat{y} + g_z \hat{z}\} q_y \hat{y} \times \hat{k} + \{g_x \hat{x} + g_y \hat{y} + g_z \hat{z}\} q_z \hat{z} \times \hat{k}\right] \quad (41)$$

$$= \left(q_x B_y \hat{z} - q_x B_z \hat{y} + q_y B_z \hat{x} - q_y B_x \hat{z} + q_z B_x \hat{y} - q_z B_y \hat{x}\right) + \left[q_k B_x \hat{k} \times \hat{x} + q_k B_y \hat{k} \times \hat{y} + q_k B_z \hat{k} \times \hat{z}\right] - i\left[\vec{g} q_x \hat{x} \times \hat{k} + \vec{g} q_y \hat{y} \times \hat{k} + \vec{g} q_z \hat{z} \times \hat{k}\right] \quad (42)$$

$$= \left(q_y B_z \hat{x} - q_z B_y \hat{x} + q_z B_x \hat{y} - q_x B_z \hat{y} + q_x B_y \hat{z} - q_y B_x \hat{z}\right) + \left[q_k B_x \hat{k} \times \hat{x} + q_k B_y \hat{k} \times \hat{y} + q_k B_z \hat{k} \times \hat{z}\right] - i\left[\vec{g} q_x \hat{x} \times \hat{k} + \vec{g} q_y \hat{y} \times \hat{k} + \vec{g} q_z \hat{z} \times \hat{k}\right] \quad (43)$$

$$= \left(q_y B_z \hat{x} - q_z B_y \hat{x} + q_z B_x \hat{y} - q_x B_z \hat{y} + q_x B_y \hat{z} - q_y B_x \hat{z}\right) + \left[q_k B_x \hat{k} \times \hat{x} + q_k B_y \hat{k} \times \hat{y} + q_k B_z \hat{k} \times \hat{z}\right] - i\left[\vec{g} q_x \hat{x} \times \hat{k} + \vec{g} q_y \hat{y} \times \hat{k} + \vec{g} q_z \hat{z} \times \hat{k}\right] \quad (44)$$

$$= \left(q_y B_z - q_z B_y\right)\hat{x} + (q_z B_x - q_x B_z)\hat{y} + \left(q_x B_y - q_y B_x\right)\hat{z} + \left[q_k B_x \hat{k} \times \hat{x} + q_k B_y \hat{k} \times \hat{y} + q_k B_z \hat{k} \times \hat{z}\right] - i\left[\vec{g} q_x \hat{x} \times \hat{k} + \vec{g} q_y \hat{y} \times \hat{k} + \vec{g} q_z \hat{z} \times \hat{k}\right] \quad (45)$$

$$= (q \times \mathrm{B})_x \hat{x} + (q \times \mathrm{B})_y \hat{y} + (q \times \mathrm{B})_z \hat{z} + \left[q_k B_x \hat{k} \times \hat{x} + q_k B_y \hat{k} \times \hat{y} + q_k B_z \hat{k} \times \hat{z}\right] - i\left[\vec{g} q_x \hat{x} \times \hat{k} + \vec{g} q_y \hat{y} \times \hat{k} + \vec{g} q_z \hat{z} \times \hat{k}\right] \quad (46)$$

$$= \vec{q} \times \vec{\mathrm{B}} + \left[q_k B_x \hat{k} \times \hat{x} + q_k B_y \hat{k} \times \hat{y} + q_k B_z \hat{k} \times \hat{z}\right] - i\left[\vec{g} q_x \hat{x} \times \hat{k} + \vec{g} q_y \hat{y} \times \hat{k} + \vec{g} q_z \hat{z} \times \hat{k}\right] \quad (47)$$

$$generalforce = \vec{f} = q \times (\nabla \times A)$$

$$= \vec{q} \times \vec{\mathrm{B}} + \left[q_k B_x \hat{k} \times \hat{x} + q_k B_y \hat{k} \times \hat{y} + q_k B_z \hat{k} \times \hat{z}\right] - i\left[\vec{g} q_x \hat{x} \times \hat{k} + \vec{g} q_y \hat{y} \times \hat{k} + \vec{g} q_z \hat{z} \times \hat{k}\right] \quad (48)$$

The result is obtained by removing the brackets step by step and the final expression for a gravitational force field is

$$\vec{f} = q \times (\nabla \times A) = \vec{q} \times \vec{\mathrm{B}} + \left[q_k B_x \hat{k} \times \hat{x} + q_k B_y \hat{k} \times \hat{y} + q_k B_z \hat{k} \times \hat{z}\right] - i\vec{g}\vec{q} \times \hat{k} \quad (49)$$

The result is obtained by removing the brackets step by step and the final expression for a gravitational force field is

$$\vec{f} = q \times (\nabla \times A) = \vec{q} \times \vec{\mathrm{B}} + \left[q_k B_x \hat{k} \times \hat{x} + q_k B_y \hat{k} \times \hat{y} + q_k B_z \hat{k} \times \hat{z}\right] - i\vec{g}\vec{q} \times \hat{k} \quad (50)$$

The general theory combines Newtonian gravity and special theory of relativity. According to Newton's second law of motion,

$$\vec{F} = ma = m\vec{g} \quad (51)$$

Therefore

$$\vec{g} = \frac{\vec{F}}{m} \quad (52)$$

this gives rise to

$$\vec{f} = q \times (\nabla \times A) = \vec{q} \times \vec{\mathrm{B}} + \left[q_k B_x \hat{k} \times \hat{x} + q_k B_y \hat{k} \times \hat{y} + q_k B_z \hat{k} \times \hat{z}\right] - i\frac{\vec{F}}{m}\vec{q} \times \hat{k} \quad (53)$$

Therefore this is the general gravitational force in the field. It is the result that this paper targeted to achieve. It contains the fully well-known Lorentz force as the real part and an additional component which now clearly specifies the nature of the gravitational field intensity. This first part is the magnetic component. The second part is the complex part which represents the Newtonian gravitation. It is clear then that gravitational force addressed using the four-vector frame work is the most accurate and detailed. The final result contains all the known information about gravity and additional information that fully specifies the nature of the gravitational force.

4. Conclusions

The real part is the magnetic component and has additional information to add on what is already known. It is clear from this result that the general gravitational field force has a magnetic component. Near a rotating mass like the earth, we have gravito-magnetic effects. These effects are what is called frame dragging effects. We also have the well-known gravitational (Newton's gravitational force) component. This kind of force is found in the complex component. This kind of force around a large rotating mass causes what is known as geodetic effects. The two effects are some of the predictions of the general theory of relativity. The additional part of the real component can now be used to explain the nature of the frame dragging effects. The exact directions can now be explained and specifically determined using this term. At this point this study has clearly from first principles shown that the gravitational field forces are best addressed within the framework of the four-dimensional complex frame. It is our suggestion that all aspects of dynamics in physics be reformulated in this mathematical framework which is by no means exhaustive.

From this result it is our belief that the troubled limited nature of the general theory of relativity can now be addressed in the four-vector frame-work. To that extend, the additional experimental information that misses out in the theory can be explicitly shown in this framework.

Our next work will aim at reformulating Ampere's law, Gauss's law, Biot-Savart law and Maxwell's field equations in the frame-work of the four vector complex frame.

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