
Prediction of Consecutive Days Maximum Rainfall Using Frequency Analysis for Nekemte Town, Oromia, Ethiopia

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Abstract: Rain is a scanty and vital hydrological factor in arid and semi-arid regions. The amount of runoff produced and rainfall received determine the development of water resources in any region. An important step in the analysis of rainfall frequency is to choose an appropriate distribution to represent the depth of rainfall to study rainfall. Analyzing the frequency of various rainfall data was attempted by Gumbel, Log normal, and Log person type III distribution method. The projected rainfall can be calculated with the aid of frequency analysis. Annual rainfall data for 22 years (2000-2021) were collected from the Ethiopian Meteorological Institute (EMI) for Nekemte station. The goal of this study is to identify the optimal theoretical probability distribution by fitting it to the maximum yearly rainfall for one day, two days, and three days distribution for the prediction of maximum annual rainfall for daily, two consecutive days, and three consecutive days. For the determination of goodness of fit chi-square, percentage absolute deviation, and the integral square error was carried out by comparing the expected values with the observed values. The results found showed that the log-normal, distribution emerged to be the best fit for the prediction of annual maximum rainfall values of Nekemte for one day. And also, another best fit was Gumbel distribution for two, and three consecutive days.

Keywords: Chi-Square, Rainfall, Gumbel, Log Normal, Log Pearson Type III

1. Introduction

Rainfall is one of the most important natural input resources to crop production and its occurrence and distribution are erratic, temporal, and spatial variations in nature [1]. Most of the hydrological events occurring as natural phenomena are observed only once. One of the important problems in hydrology deals with interpreting past records of hydrological events in terms of future probabilities of occurrence [2]. Analysis of rainfall and determination of annual maximum daily rainfall would enhance the management of water resources applications as well as the effective utilization of water resources [3]. Probability and frequency analysis of rainfall data enables us to determine the expected rainfall at various chances [4]. This information can be used to prevent floods and droughts and be applied to the planning and design of water-related engineering, such as water management, flood control work, and soil and water conservation planning [5]. Therefore, analysis of rainfall potential is necessary to

solve various water management problems and to determine crop failure due to drought or excess. Scientific prediction of rains and crop planning done analytically may prove a significant tool in the hands of farmers for better economic returns [6]. Hydraulic and design engineers require maximum daily rainfall of different return periods for safe planning and design of small and medium hydraulic structures such as small dams, bridges, culvers, and drainage works [7]. This would also be useful for forecasting the floods to downstream towns and villages [8]. Probability and frequency analysis of rainfall data enables us to determine the expected rainfall at various chances [9]. The probability distribution functions most commonly used to estimate the rainfall frequency are Gumbel, log-normal, and log-Pearson type-III distributions [10]. There is no widely accepted procedure to forecast one day, two consecutive days, and three consecutive days maximum rainfall. However, hydrological frequency analysis has an

application for predicting future events on a probability basis [11]. In the present study, an attempt was made to determine the statistical parameters and annual one day, two consecutive days, and three consecutive day's maximum rainfall using various probability levels using three probability distribution functions Gumbel, log-normal, and log-Pearson type-III distribution and to select the best probability distribution system.

2. Materials and Methods

2.1. Location and Description of the Study Area

Nekemte town is located western part of Ethiopia in the East Wallaga Zone at 328km from Addis Ababa. Nekemte has latitude and longitude of 9°2'30"N to 9°7'30"N and 36°32'00"E to 36°35' 30"E an elevation of 2080 meters and receives an annual average rainfall of 2059 mm.

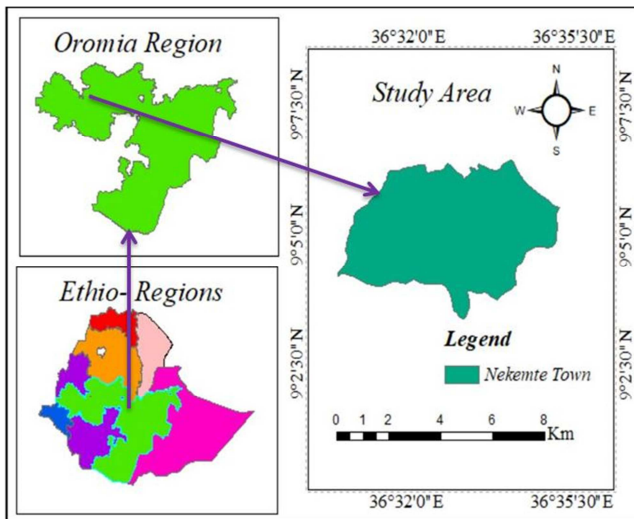


Figure 1. Location map of the study.

The annual rainfall data for 22 years (1990-2021) were collected from the Ethiopian Meteorological Institute (EMI); the extreme rainfall events for the years commencing from 1990-2021 were observed. The Gumbel, Log Normal, and Log Pearson Type III distribution methods were adopted in the analysis, which are discussed in the following subcategories.

2.2. Plotting Position Method

The purpose of frequency analysis of an annual series is to obtain a relation between the magnitude of an event and its probability of exceedance. The sample data are arranged in descending order of magnitude. Then each data is assigned an order number, m which starts from $m = 1$, for the first entry and so on till the last event for which $m = N = \text{number of years of records}$. The probability P of an event to or exceeded is given by the Weibull formula, as reported by K. Subramanya [12].

$$P = \frac{m}{N+1} \quad (1)$$

$$T = \frac{1}{P} \quad (2)$$

In which T is the recurrence interval in years.

2.3. Gumbel's Extreme Value Distribution

Gumbel's extreme value distribution was proposed by Gumbel (1941). This distribution is one of the most widely used probability distribution functions for extreme value in hydrology and meteorological studies for the prediction of maximum rainfall, maximum wind velocity, and maximum flood discharge. The most frequency distribution applicable in hydrology studies can be recommended by V. T. Chow [13] cited by [14] can be represented in equation (3):

$$XT = X + K\sigma \quad (3)$$

Where the XT is the value of the variety X of random hydrologic series with a return period T , X is the mean of the variety, σ is the standard deviation of the variety and K is the frequency factor. Equation (3) is known as the general equation of hydrologic frequency analysis. Since practical data series of extreme events of rainfall depth have a finite length of records, equation (3) is modified to account for finite N (sample size) for practical use as given below:

$$XT = X + K \cdot \sigma_{n-1} \quad (4)$$

In which σ_{n-1} represents the standard deviation of the sample and is expressed by the equation

$$\sigma_{n-1} = \sqrt{\frac{\sum (X - \bar{X})^2}{N-1}} \quad (5)$$

The frequency factor K , is expressed by the equation:

$$K = \frac{(Y_T - Y_n)}{S_n} \quad (6)$$

Where, Y_T is the reduced variety which depends on recurrence interval T and is expressed by the equation:

$$Y_T = -\left[\ln * \ln \left(\frac{T}{T-1} \right) \right] \quad (7)$$

Where Y_n is the reduced mean in equation (6) and depends upon sample size N . S_n is the reduced standard deviation in equation (6). Y_n and S_n corresponding to sample size N , are selected from tables that have been adopted by K. Subramanya [12] cited by [15].

These equations (4 to 7) are used to estimate the extreme rainfall magnitude corresponding to a given recurrence interval based on annual rainfall series:

1. The rainfall assembled for the sample size N in the present study is noted as 22 years. Annual maximum rainfall value is considered as the variety X , the mean of the variety X , and standard deviation of the sample, σ_{n-1} for the given data is calculated.
2. Using tables, the reduced mean Y_n and the reduced standard deviation S_n corresponding to the sample size, N equal to 22 are estimated. The reduced variety, Y_T for a given return period, T is computed by the equation (7).

3. The frequency factor is computed by equation (6).
4. The required value of variety X of random annual maximum rainfall series with a return period T is computed by equation (4).

2.4. Log Pearson type-III distribution

Log-Pearson type-III distribution is used in frequency analysis. In this method, the extreme rainfall magnitude of each year is first transformed into logarithmic form (base 10) and the transformed data is then analyzed. If X is the variety of random hydrologic series then the series of Z varieties where,

$$Z = \log X \quad (8)$$

For the Z series determined by the equation (8), the equation (3) for the recurrence interval can be expressed as:

$$ZT = Z + Kz \cdot \sigma_z \quad (9)$$

Where Kz is the frequency factor which depends on recurrence interval T and coefficient of skewness, Cs, Z is the mean of Z values and σ_z is the standard deviation of the Z value sample. σ_z can be expressed by the following equation:

$$\sigma_n = \sqrt{\frac{\sum (Z - \bar{Z})^2}{N-1}} \quad (10)$$

The coefficient of skewness (Cs) is expressed by the equation:

$$C_s = \frac{N \sum [Z - \bar{Z}]^3}{(N-1)(N-2)(\sigma_n)^3} \quad (11)$$

The variation of Kz corresponding to Cs and T was obtained from the table. After finding ZT using equation (9), the corresponding value of variety, XT, is obtained from the equation (12).

$$XT = \text{antilog } ZT \quad (12)$$

The computation of the theoretical rainfall magnitudes corresponding to different recurrence intervals was determined [2].

2.5. Log Normal Distribution

Log Normal distribution is a special case of Log Pearson type III distribution in which the coefficient of skewness (Cs) is zero. The other statistics like Z are calculated for the transformed rainfall data through equation (8) and σ_z can be calculated from equation (10), the values of Kz for a given return period T and Cs = 0 are read from table [16] cited by [17]. Extreme rainfall values are estimated through equation (9).

2.6. Goodness of Fit Criteria

2.6.1. Chi-Square Test

This test is applicable to various problems of hydro-

meteorological nature. It is primarily used for testing the agreement of the observed data with those expected upon a given hypothesis [1, 2, 10, 11]. The Chi-Square values, χ^2 can be calculated by [18]:

$$\chi^2 = \frac{(R_o - R_e)^2}{R_e} \quad (13)$$

Ro and Re are the observed and estimated rainfall magnitudes, respectively. The distribution with the least average of the Chi-Square values is adjudged to be the best [19]. The $\chi^2 = 0$ indicates the Ro and Re rainfall magnitudes agree exactly. The χ^2 values for each distribution are shown in Tables 4, 10, and 16.

2.6.2. Percentage Absolute Deviation

In order to test the goodness of fit of the computed and observed rainfall magnitudes, percentage absolute deviations (PAD) is recommended by [1, 2] and is determined by the equation which can be expressed as:

$$PAD = \frac{|R_o - R_e|}{R_o} * 100\% \quad (14)$$

Where, PAD is the percentage absolute deviation of the computed extreme rainfall values with respect to the observed values given in Tables 5, 11, and 17.

2.6.3. Integral Square Error

The integral square error (I. S. E) was used to measure the goodness of fit between the observed and estimated extreme rainfall. The integral square error values of distribution were estimated as reported by [1, 2].

$$ISE = \frac{[\sum_{i=1}^m (R_{ei} - R_{oi})^2]^{0.5}}{\sum_{i=1}^m R_{oi}} \quad (15)$$

Where Roi, and Rei are the observed values of the estimated extreme rainfall magnitudes concerning the observed values are given in Tables 6, 12, and 18.

Analysis of consecutive day's maximum rainfall at different return periods is a basic tool for safe and economical planning and design of small dams, bridges, culverts, irrigation and drainage work, etc. Though the nature of rainfall is erratic and varies with time and space, it is possible to predict design rainfall fairly accurately for certain return periods using various probability distributions. The results of this study have been discussed in this section.

Frequency analysis is used to predict how often certain values of a variable phenomenon may occur and to assess the reliability of the prediction. It is a tool for determining design rainfalls and design discharges for drainage works and drainage structures, especially about their required hydraulic capacity. Three different distributions were used to fit the observed maximum rainfall data for daily, two consecutive days, and three consecutive days.

3. Results & Discussion

3.1. One Day Annual Maximum Rainfall

Table 1. Estimation of one day maximum rainfall using Gumbel distribution.

| $\bar{X}=83$ | $Y_n=0.5268$ | $S_n=1.0754$ | $\sigma_{n-1}=22$ |
|-------------------------|---|--|--|
| Return period (T), year | Reduced variety $Y_T = -\ln \ln \left[\frac{T}{T-1} \right]$ | Frequency factor $K = \frac{Y_T - \bar{Y}_n}{S_n}$ | Estimated rainfall $X_T = \bar{X} + K\sigma_{n-1}$ |
| 5 | 1.4999 | 0.9049 | 102.907 |
| 10 | 2.2504 | 1.6028 | 118.261 |
| 15 | 2.6738 | 1.9965 | 126.922 |
| 25 | 3.1985 | 2.4844 | 137.656 |
| 50 | 3.9019 | 3.1385 | 152.046 |
| 75 | 4.3108 | 3.5187 | 160.411 |
| 100 | 4.6002 | 3.7878 | 166.222 |

Table 2. Estimation of one day maximum rainfall using Log Normal distribution.

| Return period (T) | $\bar{Z}=1.906$ | $\sigma_z=0.109$ | $C_s=0$ |
|-------------------|-----------------|-----------------------------|-----------------|
| Year | KZ (From Table) | $Z_T = \bar{Z} + K\sigma_z$ | XT = antilog ZT |
| 5 | 0.842 | 1.998 | 99.541 |
| 10 | 1.282 | 2.046 | 111.173 |
| 15 | 1.722 | 2.094 | 124.165 |
| 25 | 1.751 | 2.097 | 125.026 |
| 50 | 2.054 | 2.130 | 134.896 |
| 75 | 2.190 | 2.145 | 139.637 |
| 100 | 2.326 | 2.160 | 144.544 |

Table 3. Estimation of one-day maximum rainfall using Log Pearson type-III distribution.

| Return period (T) | $\bar{Z}=1.906$ | $\sigma_z=0.109$ | $C_s=0.5$ |
|-------------------|-----------------|-----------------------------|-----------------|
| Year | KZ (From Table) | $Z_T = \bar{Z} + K\sigma_z$ | XT = antilog ZT |
| 5 | 0.808 | 1.994 | 98.628 |
| 10 | 1.323 | 2.050 | 112.202 |
| 15 | 1.519 | 2.072 | 118.032 |
| 25 | 1.910 | 2.114 | 130.017 |
| 50 | 2.311 | 2.158 | 143.880 |
| 75 | 2.499 | 2.178 | 150.661 |
| 100 | 2.686 | 2.199 | 158.125 |

Table 4. Chi-Square test for goodness of fit for theoretical probability distribution for one day annual maximum rainfall data of Nekemte.

| Return period (T) | Probability (P) | Ro | Expected rainfall (Re) | | | $\chi^2 = \frac{(R_o - R_e)^2}{R_e}$ | | |
|-------------------|-----------------|-------|------------------------|---------|---------|--------------------------------------|-------|-------|
| year | % | mm | GD | LND | LPT3D | GD | LND | LPT3D |
| 5 | 20 | 89 | 102.907 | 99.541 | 98.628 | 1.879 | 1.116 | 0.940 |
| 10 | 10 | 91.8 | 118.261 | 111.173 | 112.202 | 5.921 | 3.376 | 3.710 |
| 15 | 6.7 | 104.2 | 126.922 | 124.165 | 118.032 | 4.068 | 3.210 | 1.621 |
| 25 | 4 | 105.5 | 137.656 | 125.026 | 130.017 | 7.512 | 3.049 | 4.623 |
| 50 | 2 | 110.6 | 152.046 | 134.896 | 143.880 | 11.298 | 4.376 | 7.698 |
| 75 | 1.3 | 119 | 160.411 | 139.637 | 150.661 | 10.690 | 3.050 | 6.653 |
| 100 | 1 | 137.5 | 166.222 | 144.544 | 158.125 | 4.963 | 0.343 | 2.690 |
| Mean | | | | | | 6.624 | 2.646 | 3.991 |

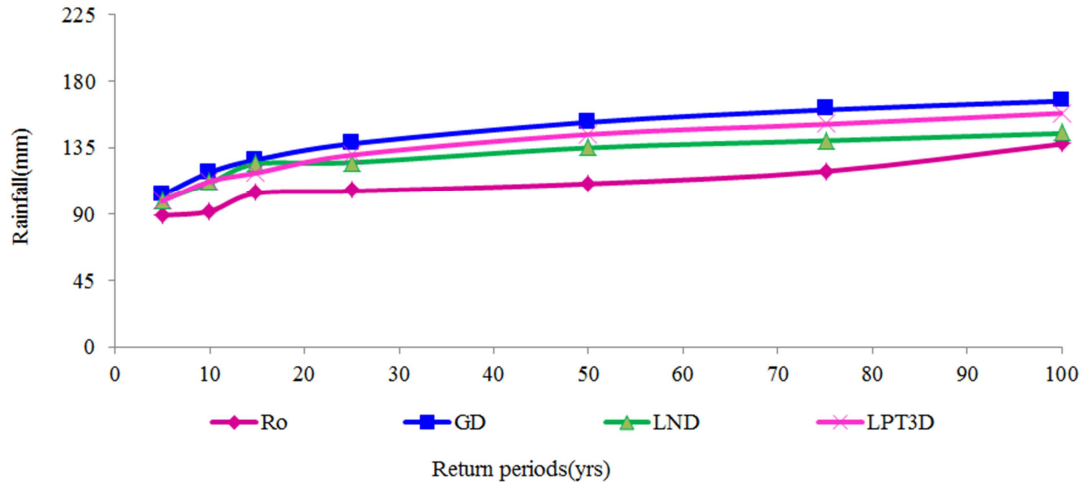
Key: Ro – Observed rainfall, GD – Gumbel distribution, LND – Log normal distribution, and LPT3D – Log Pearson type three distributions.

Table 5. Percent Absolute Deviation values for the goodness of fit for theoretical probability distribution for one day annual maximum rainfall data of Nekemte.

| Return period (T) | Probability (P) | Ro | Expected rainfall (Re) | | | $PAD = \frac{ R_o - R_e }{R_o} * 100\%$ | | |
|-------------------|-----------------|-------|------------------------|---------|---------|---|--------|--------|
| Year | % | mm | GD | LND | LPT3D | GD | LND | LPT3D |
| 5 | 20 | 89 | 102.907 | 99.541 | 98.628 | 15.626 | 11.844 | 10.818 |
| 10 | 10 | 91.8 | 118.261 | 111.173 | 112.202 | 28.825 | 21.103 | 22.224 |
| 15 | 6.7 | 104.2 | 126.922 | 124.165 | 118.032 | 21.806 | 19.160 | 13.274 |
| 25 | 4 | 105.5 | 137.656 | 125.026 | 130.017 | 30.480 | 18.508 | 23.239 |
| 50 | 2 | 110.6 | 152.046 | 134.896 | 143.880 | 37.474 | 21.967 | 30.090 |
| 75 | 1.3 | 119 | 160.411 | 139.637 | 150.661 | 34.799 | 17.342 | 26.606 |
| 100 | 1 | 137.5 | 166.222 | 144.544 | 158.125 | 20.969 | 5.123 | 15.000 |
| Mean | | | | | | 27.140 | 16.435 | 20.179 |

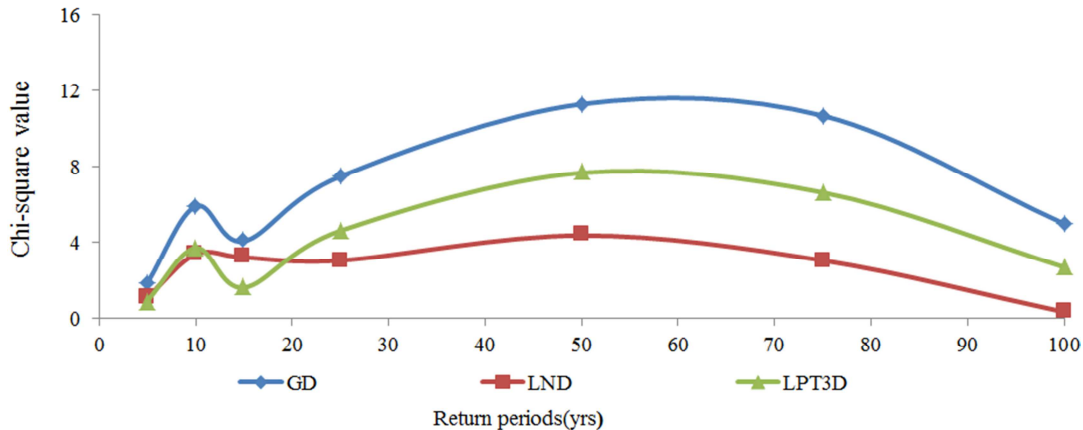
Table 6. Integral Square Error values for the goodness of fit for theoretical probability distribution for one day annual maximum rainfall data of Nekemte.

| Return period (T) | Probability (P) | Ro | Expected rainfall (Re) | | | I. S. E = $\left[\frac{\sum_{i=1}^m (R_{ei} - R_{oi})^2}{\sum_{i=1}^m R_{oi}} \right]^{\frac{1}{2}}$ | | |
|-------------------|-----------------|-------|------------------------|---------|---------|---|-------|-------|
| Year | % | mm | GD | LND | LPT3D | GD | LND | LPT3D |
| 5 | 20 | 89 | 102.907 | 99.541 | 98.628 | 0.108 | 0.064 | 0.082 |
| 10 | 10 | 91.8 | 118.261 | 111.173 | 112.202 | | | |
| 15 | 6.7 | 104.2 | 126.922 | 124.165 | 118.032 | | | |
| 25 | 4 | 105.5 | 137.656 | 125.026 | 130.017 | | | |
| 50 | 2 | 110.6 | 152.046 | 134.896 | 143.880 | | | |
| 75 | 1.3 | 119 | 160.411 | 139.637 | 150.661 | | | |
| 100 | 1 | 137.5 | 166.222 | 144.544 | 158.125 | | | |

**Figure 2.** Observed and estimated one day annual maximum rainfall.

The observed and computed values for one day maximum annual rainfall obtained by using Gumbel, Lognormal, and Log Pearson type-III distributions were plotted in Figure 2 based on the computations presented in tabular form in Tables 1 through 3. It is clear from Figure 2 above that the observed one-day annual maximum rainfall is very close to the theoretical values using Log normal distribution. The best probability distribution was adjudged by comparing the average of Chi-Square, Percentage absolute deviation (PAD), and Integral square error (I. S. E.) values in percent obtained for these distributions corresponding to return periods 5, 10, 15, 25, 50, 75 and 100 years respectively as shown in Tables 4

through 6. The average of Chi-square values for Gumbel, Lognormal, and Log Pearson type-III was found to be 6.624, 2.646, and 3.991 respectively. The average of PAD values for Gumbel, Lognormal, and Log Pearson type-III distributions were observed to be 27.140, 16.435, and 20.179 respectively and values of I. S. E for Gumbel, Lognormal, and Log Pearson type-III distributions were 0.108, 0.064, and 0.082. Hence, Log normal Distribution gives the best fit for the predicted one-day annual maximum rainfall values for Nekemte based on performance evaluation criteria as shown in Figure 3 below.

**Figure 3.** Observed and estimated one day annual maximum rainfall.

3.2. Two Days Consecutive Maximum Annual Rainfall

Table 7. Estimation of two consecutive day's maximum rainfall using Gumbel distribution.

| $\bar{X}=104$ | $Y_n=0.5268$ | $S_n=1.0754$ | $\sigma_{n-1}=24$ |
|------------------------|---|--|--|
| Return period (T) Year | Reduced variety $Y_T = -\ln \ln \left[\frac{T}{T-1} \right]$ | Frequency factor $K = \frac{Y_T - Y_n}{S_n}$ | Estimated rainfall $X_T = \bar{X} + K\sigma_{n-1}$ |
| 5 | 1.4999 | 0.9049 | 125.718 |
| 10 | 2.2504 | 1.6028 | 142.467 |
| 15 | 2.6738 | 1.9965 | 151.916 |
| 25 | 3.1985 | 2.4844 | 163.626 |
| 50 | 3.9019 | 3.1385 | 179.324 |
| 75 | 4.3108 | 3.5187 | 188.449 |
| 100 | 4.6002 | 3.7878 | 194.907 |

Table 8. Estimation of two day maximum rainfall using Log Normal distribution.

| Return period (T) | $\bar{Z}=2.006$ | $\sigma_n=0.139$ | $C_s=0$ |
|-------------------|-----------------|-----------------------------|-----------------|
| Year | KZ (From Table) | $Z_T = \bar{Z} + K\sigma_z$ | XT = antilog ZT |
| 5 | 0.842 | 2.123 | 132.739 |
| 10 | 1.282 | 2.184 | 152.757 |
| 15 | 1.722 | 2.245 | 175.792 |
| 25 | 1.751 | 2.249 | 177.419 |
| 50 | 2.054 | 2.292 | 195.885 |
| 75 | 2.190 | 2.310 | 204.174 |
| 100 | 2.326 | 2.329 | 213.305 |

Table 9. Estimation of two-day maximum rainfall using Log Pearson type-III distribution.

| Return period (T) | $\bar{Z}=2.006$ | $\sigma_z=0.139$ | $C_s=1.3$ |
|-------------------|-----------------|-----------------------------|-----------------|
| Year | KZ (From Table) | $Z_T = \bar{Z} + K\sigma_z$ | XT = antilog ZT |
| 5 | 0.719 | 2.106 | 127.644 |
| 10 | 1.229 | 2.192 | 155.597 |
| 15 | 1.595 | 2.228 | 169.044 |
| 25 | 2.108 | 2.299 | 199.067 |
| 50 | 2.666 | 2.377 | 238.232 |
| 75 | 2.939 | 2.415 | 260.016 |
| 100 | 3.211 | 2.452 | 283.139 |

Table 10. Chi-Square test for the goodness of fit for theoretical probability distribution for two day annual maximum rainfall data of Nekemte.

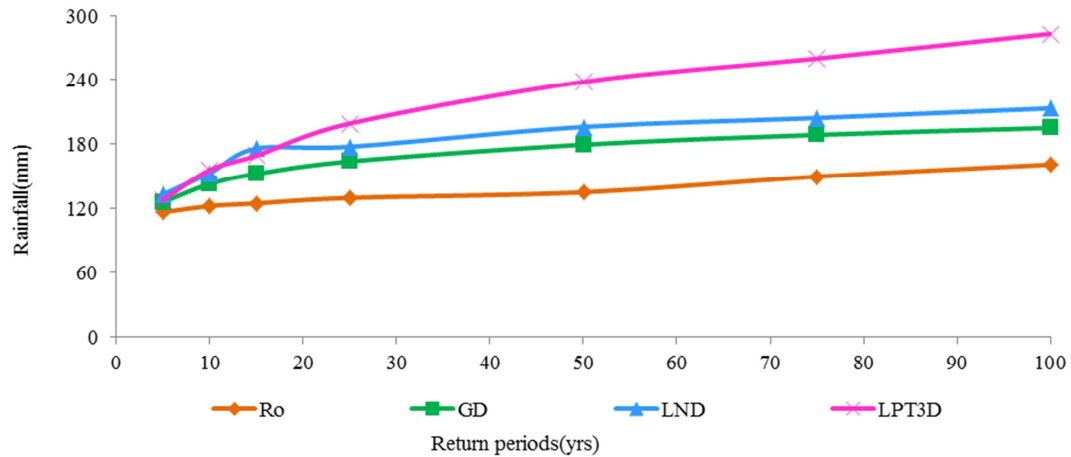
| Return period (T) | Probability (P) | Ro | Expected rainfall (Re) | | | $\chi^2 = \frac{(R_o - R_e)^2}{R_e}$ | | |
|-------------------|-----------------|-------|------------------------|---------|---------|--------------------------------------|--------|--------|
| year | % | mm | GD | LND | LPT3D | GD | LND | LPT3D |
| 5 | 20 | 116.2 | 125.718 | 132.739 | 127.644 | 0.721 | 2.061 | 1.026 |
| 10 | 10 | 122 | 142.467 | 152.757 | 155.597 | 2.940 | 6.193 | 7.254 |
| 15 | 6.7 | 124.5 | 151.916 | 175.792 | 169.044 | 4.948 | 14.966 | 11.738 |
| 25 | 4 | 129.5 | 163.626 | 177.419 | 199.067 | 7.117 | 12.942 | 24.311 |
| 50 | 2 | 134.7 | 179.324 | 195.885 | 238.232 | 11.104 | 19.111 | 44.993 |
| 75 | 1.3 | 149.4 | 188.449 | 204.174 | 260.016 | 8.091 | 14.694 | 47.058 |
| 100 | 1 | 160.7 | 194.907 | 213.305 | 283.139 | 6.003 | 12.973 | 52.947 |
| Mean | | | | | | 5.846 | 11.849 | 27.047 |

Table 11. Percent Absolute Deviation values for the goodness of fit for theoretical probability distribution for two day annual maximum rainfall data of Nekemte.

| Return period (T) | Probability (P) | Ro | Expected rainfall (Re) | | | $PAD = \frac{ R_o - R_e }{R_o} * 100\%$ | | |
|-------------------|-----------------|-------|------------------------|---------|---------|---|--------|--------|
| Year | % | mm | GD | LND | LPT3D | GD | LND | LPT3D |
| 5 | 20 | 116.2 | 125.718 | 132.739 | 127.644 | 8.191 | 14.222 | 9.849 |
| 10 | 10 | 122 | 142.467 | 152.757 | 155.597 | 16.776 | 25.211 | 27.539 |
| 15 | 6.7 | 124.5 | 151.916 | 175.792 | 169.044 | 22.021 | 41.198 | 35.778 |
| 25 | 4 | 129.5 | 163.626 | 177.419 | 199.067 | 26.352 | 37.003 | 53.720 |
| 50 | 2 | 134.7 | 179.324 | 195.885 | 238.232 | 22.128 | 45.423 | 76.861 |
| 75 | 1.3 | 149.4 | 188.449 | 204.174 | 260.016 | 26.137 | 36.663 | 74.040 |
| 100 | 1 | 160.7 | 194.907 | 213.305 | 283.139 | 21.286 | 32.735 | 76.191 |
| Mean | | | | | | 21.984 | 22.209 | 50.568 |

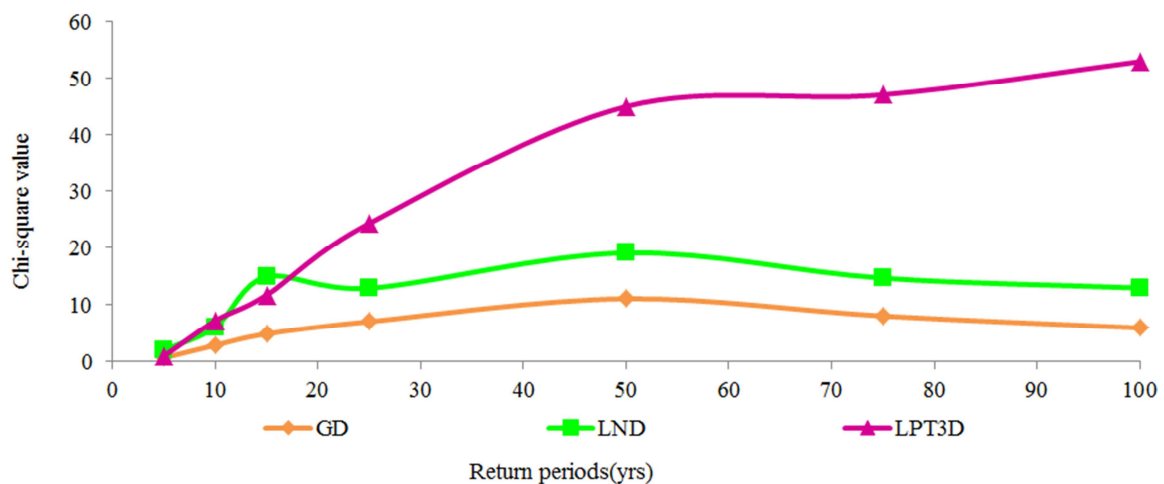
Table 12. Integral Square Error values for the goodness of fit for theoretical probability distribution for two day annual maximum rainfall data of Nekemte.

| Return period (T) | Probability (P) | RO | Expected rainfall (Re) | | | I. S. E = $\left[\frac{\sum_{i=1}^m (R_{ei} - R_{oi})^2}{\sum_{i=1}^m R_{oi}} \right]^{\frac{1}{2}}$ | | |
|-------------------|-----------------|-------|------------------------|---------|---------|---|-------|-------|
| Year | % | mm | GD | LND | LPT3D | GD | LND | LPT3D |
| 5 | 20 | 116.2 | 125.718 | 132.739 | 127.644 | 0.123 | 0.167 | 0.272 |
| 10 | 10 | 122 | 142.467 | 152.757 | 155.597 | | | |
| 15 | 6.7 | 124.5 | 151.916 | 175.792 | 169.044 | | | |
| 25 | 4 | 129.5 | 163.626 | 177.419 | 199.067 | | | |
| 50 | 2 | 134.7 | 179.324 | 195.885 | 238.232 | | | |
| 75 | 1.3 | 149.4 | 188.449 | 204.174 | 260.016 | | | |
| 100 | 1 | 160.7 | 194.907 | 213.305 | 283.139 | | | |

**Figure 4.** Observed and estimated two day annual maximum rainfall.

The observed and computed values for three-day maximum annual rainfall obtained by using Gumbel, Lognormal, and Log Pearson type-III distributions were plotted in Figure 4 based on the computations presented in tabular form in Tables 7 through 9. It is clear from Figure 4 above that the observed three-day annual maximum rainfall is very close to the theoretical values using the Gumbel distribution. The best probability distribution was adjudged by comparing the average of Chi-Square, Percentage absolute deviation (PAD), and Integral square error (I. S. E.) values in percent obtained for these distributions corresponding to return periods 5, 10, 15, 25, 50, 75 and 100 years respectively

as shown in Tables 10 through 12. The average of Chi-square values for Gumbel, Lognormal, and Log Pearson type-III was found to be 5.846, 11.849, and 27.047, respectively. The average of PAD values for Gumbel, Lognormal, and Log Pearson type-III distributions was observed to be 21.984, 22.209, and 50.568 respectively and values of I. S. E. for Gumbel, Lognormal, and Log Pearson type-III distributions were 0.123, 0.167, and 0.272. Hence, Gumbel Distribution gives the best fit for the predicted two-day annual maximum rainfall values for Nekemte based on performance evaluation criteria as indicated in Figure 5 below.

**Figure 5.** Observed and estimated two day annual maximum rainfall.

3.3. Three Days Consecutive Annual Maximum Rainfall

Table 13. Estimation of three consecutive day's maximum rainfall using Gumbel distribution.

| $\bar{X} = 134$ | $Y_n = 0.5268$ | $S_n = 1.0754$ | $\sigma_{n-1} = 26$ |
|-------------------------|--|---|---|
| Return period (T), year | Reduced variety $Y_T = -\ln \ln \left[\frac{T}{T-1} \right]$ | Frequency factor $K = \frac{Y_T - Y_n}{S_n}$ | Estimated rainfall $X_T = \bar{X} + K\sigma_{n-1}$ |
| 5 | 1.4999 | 0.9049 | 155.718 |
| 10 | 2.2504 | 1.6028 | 172.467 |
| 15 | 2.6738 | 1.9965 | 181.916 |
| 25 | 3.1985 | 2.4844 | 193.626 |
| 50 | 3.9019 | 3.1385 | 209.324 |
| 75 | 4.3108 | 3.5187 | 218.449 |
| 100 | 4.6002 | 3.7878 | 224.907 |

Table 14. Estimation of three day maximum rainfall using Log Normal distribution.

| Return period (T) | $\bar{Z} = 2.119$ | $\sigma_n = 0.222$ | $C_s = 0$ |
|-------------------|-------------------|------------------------------|-----------------|
| Year | KZ (From Table) | $Z_T = \bar{Z} + Kz\sigma_z$ | XT = antilog ZT |
| 5 | 0.842 | 2.315 | 206.538 |
| 10 | 1.282 | 2.418 | 261.818 |
| 15 | 1.722 | 2.520 | 331.131 |
| 25 | 1.751 | 2.527 | 336.512 |
| 50 | 2.054 | 2.598 | 396.278 |
| 75 | 2.190 | 2.629 | 425.598 |
| 100 | 2.326 | 2.661 | 458.142 |

Table 15. Estimation of three-day maximum rainfall using Log Pearson type-III distribution.

| Return period (T) | $\bar{Z} = 2.119$ | $\sigma_z = 0.222$ | $C_s = 1.1$ |
|-------------------|-------------------|------------------------------|-----------------|
| Year | KZ (From Table) | $Z_T = \bar{Z} + Kz\sigma_z$ | XT = antilog ZT |
| 5 | 0.745 | 2.293 | 196.149 |
| 10 | 1.341 | 2.431 | 270.055 |
| 15 | 1.583 | 2.488 | 307.496 |
| 25 | 2.066 | 2.600 | 398.454 |
| 50 | 2.585 | 2.721 | 526.387 |
| 75 | 2.836 | 2.780 | 602.266 |
| 100 | 3.087 | 2.838 | 689.082 |

Table 16. Chi-Square test for the goodness of fit for theoretical probability distribution for three day annual maximum rainfall data of Nekemte.

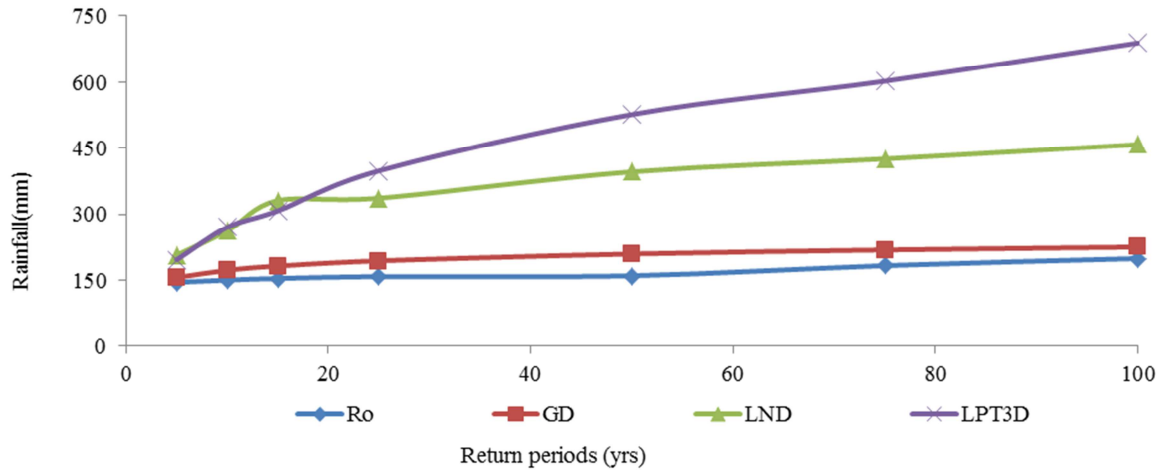
| Return period (T) | Probability (P) | Ro | Expected rainfall (Re) | | | $\chi^2 = \frac{(R_o - R_e)^2}{R_e}$ | | |
|-------------------|-----------------|-------|------------------------|---------|---------|--------------------------------------|---------|---------|
| year | % | mm | GD | LND | LPT3D | GD | LND | LPT3D |
| 5 | 20 | 145.2 | 155.718 | 206.538 | 196.149 | 0.710 | 18.216 | 13.234 |
| 10 | 10 | 150.1 | 172.467 | 261.818 | 270.055 | 2.901 | 47.670 | 53.282 |
| 15 | 6.7 | 153.9 | 181.916 | 221.131 | 307.496 | 4.315 | 94.859 | 76.722 |
| 25 | 4 | 158 | 193.626 | 226.512 | 398.454 | 6.555 | 94.697 | 145.106 |
| 50 | 2 | 159.9 | 209.324 | 396.278 | 526.387 | 11.670 | 140.998 | 255.160 |
| 75 | 1.3 | 182.9 | 218.449 | 425.598 | 602.266 | 5.785 | 138.399 | 292.010 |
| 100 | 1 | 199 | 224.907 | 458.142 | 689.082 | 2.984 | 146.580 | 348.551 |
| Mean | | | | | | 4.989 | 97.346 | 169.152 |

Table 17. Percent Absolute Deviation values for the goodness of fit for theoretical probability distribution for three day annual maximum rainfall data of Nekemte.

| Return period (T) | Probability (P) | Ro | Expected rainfall (Re) | | | $PAD = \frac{ R_o - R_e }{R_o} * 100\%$ | | |
|-------------------|-----------------|-------|------------------------|---------|---------|---|---------|---------|
| year | % | mm | GD | LND | LPT3D | GD | LND | LPT3D |
| 5 | 20 | 145.2 | 155.718 | 206.538 | 196.149 | 7.244 | 42.244 | 35.089 |
| 10 | 10 | 150.1 | 172.467 | 261.818 | 270.055 | 14.901 | 74.429 | 79.917 |
| 15 | 6.7 | 153.9 | 181.916 | 221.131 | 307.496 | 18.204 | 115.160 | 99.802 |
| 25 | 4 | 158 | 193.626 | 226.512 | 398.454 | 22.548 | 112.982 | 152.186 |
| 50 | 2 | 159.9 | 209.324 | 396.278 | 526.387 | 30.909 | 147.829 | 229.198 |
| 75 | 1.3 | 182.9 | 218.449 | 425.598 | 602.266 | 19.436 | 132.694 | 229.287 |
| 100 | 1 | 199 | 224.907 | 458.142 | 689.082 | 13.019 | 130.222 | 246.272 |
| Mean | | | | | | 18.037 | 107.937 | 153.107 |

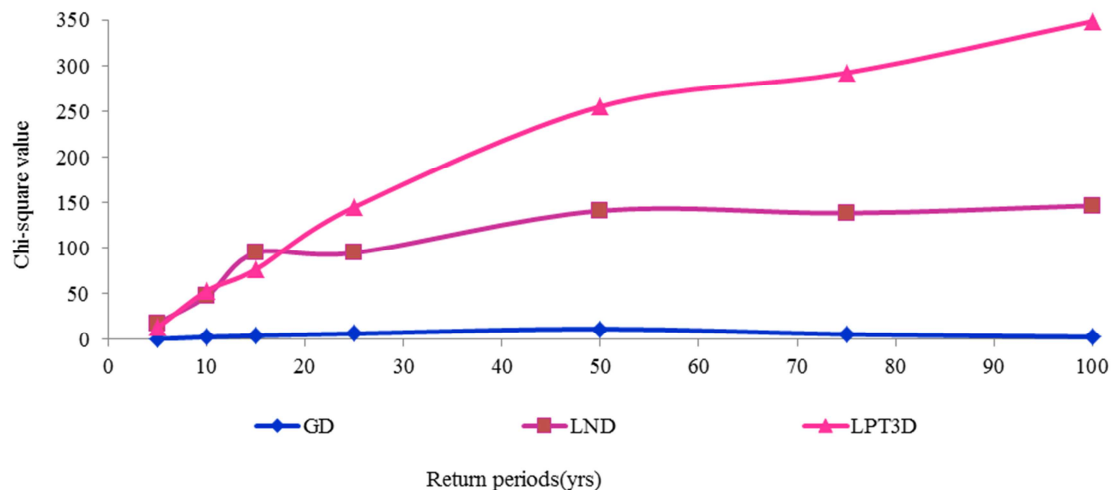
Table 18. Integral Square Error values for the goodness of fit for theoretical probability distribution for three day annual maximum rainfall data of Nekemte.

| Return period (T) | Probability (P) | Ro | Expected rainfall (Re) | | | I. S. E = $\frac{[\sum_{i=1}^m (R_{ei} - R_{oi})^2]^{\frac{1}{2}}}{\sum_{i=1}^m R_{oi}}$ | | |
|-------------------|-----------------|-------|------------------------|---------|---------|--|-------|-------|
| year | % | mm | GD | LND | LPT3D | GD | LND | LPT3D |
| 5 | 20 | 145.2 | 155.718 | 206.538 | 196.149 | 0.073 | 0.445 | 0.701 |
| 10 | 10 | 150.1 | 172.467 | 261.818 | 270.055 | | | |
| 15 | 6.7 | 153.9 | 181.916 | 221.131 | 307.496 | | | |
| 25 | 4 | 158 | 193.626 | 226.512 | 398.454 | | | |
| 50 | 2 | 159.9 | 209.324 | 396.278 | 526.387 | | | |
| 75 | 1.3 | 182.9 | 218.449 | 425.598 | 602.266 | | | |
| 100 | 1 | 199 | 224.907 | 458.142 | 689.082 | | | |

**Figure 6.** Observed and estimated three day annual maximum rainfall.

The observed and computed values for three-day maximum annual rainfall obtained by using Gumbel, Lognormal, and Log Pearson type-III distributions were plotted in Figure 6 based on the computations presented in tabular form in Tables 13 through 15. It is clear from Figure 6 above that the observed three-day annual maximum rainfall is very close to the theoretical values using the Gumbel distribution. The best probability distribution was adjudged by comparing the average of Chi-Square, Percentage absolute deviation (PAD), and Integral square error (I. S. E.) values in percent obtained for these distributions corresponding to return periods 5, 10, 15, 25, 50, 75 and 100 years respectively

as shown in Tables 16 through 18. The average of Chi-square values for Gumbel, Lognormal, and Log Pearson type-III was found to be 4.989, 97.346, and 169.152 respectively. The average of PAD values for Gumbel, Lognormal, and Log Pearson type-III distributions were observed to be 18.037, 107.937, and 153.107 respectively and values of I. S. E for Gumbel, Lognormal, and Log Pearson type-III distributions were 0.073, 0.445, and 0.701. Hence, Gumbel Distribution gives the best fit for the predicted three-day annual maximum rainfall values for Nekemte based on performance evaluation criteria as shown in Figure 7 below.

**Figure 7.** Observed and estimated three day annual maximum rainfall.

4. Conclusion

Rainfall is a renewable resource, highly variable in space and time, and subject to depletion or enhancement due to both natural and anthropogenic causes. The frequency analysis of annual one-day, two days, and three days maximum rainfall for identifying the best-fit probability distributions can be studied for three probability distributions such as Gumbel's, Log Normal, and Pearson Type-III by using Chi-square goodness, Percentage absolute deviation, and Integral square error of fit test. The results of the study were the mean, standard deviation, coefficient of variation, and coefficients of skewness were 83mm, 22, 0.109, and 0.5 for one-day maximum rainfall respectively. Also, the mean, standard deviation, coefficient of variation, and coefficient of skewness were 104mm, 24, 0.139, and 1.3 for two days of maximum rainfall respectively. Finally, the mean, standard deviation, coefficient of variation, and coefficient of skewness were 134mm, 26, 0.222, and 1.1 for three days of maximum rainfall respectively. This study gives an idea about the prediction of annual one-day, two days, and three-day maximum rainfall to design small and medium hydraulic structures, soil and water conservation structures, irrigation structures, and drainage works.

Data Availability

All information provided to this publication is presented in the full document.

Conflict of Interest

The author declares that they have no conflicts of interest.

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